A SELF-CONSISTENT APPROACH TO THE LASER COOLING OF V-TYPE ATOMS

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ABSTRACT
The equations of motion for three energy level V-type atoms driven by two counter propagating laser fields are derived from Schrödinger's equation. These equations at steady state are reduced to a single integral equation for the ground state momentum distribution. A numerical method, based on a self-consistent approach, is developed to study the laser cooling of V-type atoms. A good agreement is reached between our numerical results and those from existing theory. [1]
[1] Y. Castin, H. Wallis, and J. Dalibard, J. Opt. Am., B 6, (1989), p. 2046.

INTRODUCTION

A single photon has a momentum $\hbar k$ and has an energy $\hbar\omega$, where $\hbar$ is Planck's constant divided by 2π and k is the wave number and ω is the frequency of the electromagnetic field of the photon. Due to momentum conservation, an atom of mass M and momentum p can gain or lose momentum of $\hbar k$ during the absorption or emission of a single photon. Consequently, the kinetic energy of the atom changes by:

$$\frac{(p \pm \hbar k)^2}{2M} - \frac{p^2}{2M} = \frac{\pm \hbar k p}{M} + E_r \,, \qquad (1)$$

where kp/M is the Doppler frequency shift and E_r is the atomic recoil energy:

$$E_r = \frac{(\hbar k)^2}{2M} = \hbar\omega_r \,, \qquad (2)$$

were ω_r is called the recoil frequency shift. These ideas form the basis for the laser cooling of atoms. [1]

Temperature is a measure of the average kinetic energy of the atoms in a system. The average kinetic energy is determined by the atomic momentum (or velocity) distribution. Atoms in thermal equilibrium follow the Maxwell-Boltzmann distribution; the broader the distribution the higher the temperature. The essential task of laser cooling is to narrow the momentum distribution profile by an exchange of momentum between atoms and photons. The simplest laser cooling scheme is when atoms of two energy levels are driven by two opposing laser fields of the same frequency. The Doppler effect causes the atoms to see the laser beam traveling in the direction opposite to their motion to have a higher frequency than that along their direction of motion. If the laser frequency is less than the atomic transition frequency and the motion of the atoms are slow enough, the frequency shift due to the Doppler effect will always bring the energy of the counter propagating laser closer to energy of the atomic transition than the copropagating one. As a result, the radiation force

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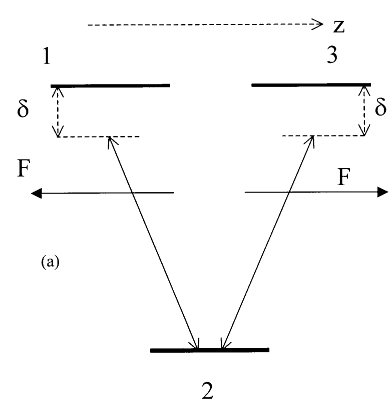

Figure 1
A schematic diagram of three-level 'V' type atoms driven by two counter propagating laser fields of amplitude F, and frequency detuning δ. The system is oriented along the z axis.

given to the atom by counter propagating laser is always larger than that given by the copropagating laser. This effect gives rise to a net force opposing the motion of the atoms, and results in the narrowing of the momentum distribution of the atoms and the reduction of the atomic temperature.

A V-type atom, the kind that is discussed in this paper, is a three energy level atom whose levels are arranged in a 'V' as shown in Figure 1. The transitions between the quantum states 1 -> 2 and 3 -> 2 are driven independently by two counter propagating laser fields of the same amplitude F, frequency ω and wave number k. We are interested in momentum distribution of the 'cooled' atoms, both in the broad distribution ($\Delta p >> \hbar k$) and narrow distribution ($\Delta p \approx \hbar k$), where Δp is the width of the atomic momentum distribution. In the case of the narrow momentum distribution, the deBroglie wavelength of the atom, $\lambda_D = h/p \approx h/\hbar k = 2\pi/k = \lambda$ is on the order of the wavelength of the laser field. In this case, the atoms can no longer be viewed as localized particles moving classically under the electromagnetic fields of the lasers.

A more accurate theoretical description requires the simultaneous quantization of both the internal and external (center-of-mass) degrees of freedom. In a theory developed for the 'V' system, the steady-state momentum distribution is obtained either by integrating the generalized Optical Bloch (GOB) equation over a long period of time or solving directly the steady-state coupled GOB equations. [2] The former is very time consuming, while the latter involves matrices of large dimension. In this work, we approach this problem in a self-consistent manner. In a self-consistent approach, we start from a guessed function and use the equation of interest to arrive at an improved solution in an iterative manner until the solution does not appear to change with further iteration. This method is easy to implement and quite efficient for atoms with narrow atomic lines.

EQUATIONS OF MOTION

The time dependent Schrödinger equation for this system is:

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \left(\hat{H}_A + \frac{1}{2}\frac{\hat{p}^2}{M} + \hat{H}_{AL}\right)|\psi\rangle , \tag{3}$$

where $\hat{H}_A$ represents the interaction among the particles inside the atom, $\frac{1}{2}\frac{\hat{p}^2}{M}$ is the kinetic energy of the center-of-mass and $\hat{H}_{AL}$ is the dipole interaction between the atom and the laser fields. The interaction of the atoms with the vacuum is not included in Equation 3 but will be introduced later phenomenologically. We choose to work in a space spanned by $|i,p\rangle$, defined as:

$$\hat{H}_A\,|i,p\rangle = \hbar\,\Omega_{i2}\,|i,p\rangle$$
$$\hat{p}\,|i,p\rangle = p\,|i,p\rangle \quad , \tag{4}$$

where $\hbar\,\Omega_{i2}$ is the energy of the i^{th} level relative to the ground state energy and p is the eigenvalue of the momentum operator. The system is closed in the sense that:

$$\int_{-\infty}^{+\infty}\sum_{n=1}^{3}|i,p\rangle\langle i.p|\,dp = 1\ . \tag{5}$$

The dipole interaction is given by:

$$\hat{H}_{AL} = -\hat{\mu}\cdot\hat{\boldsymbol{F}}\ , \tag{6}$$

where the electric dipole momentum operator is:

$$\hat{\mu} = e\,\hat{\boldsymbol{r}}_{\boldsymbol{e}}\ , \tag{7}$$

where e is the charge of the electron and $\hat{\boldsymbol{r}}_{\boldsymbol{e}}$ is the position operator for the electron, and the total field operator is given by

$$\hat{F}(\hat{z},t) = \frac{1}{2}\left(\vec{e}_{32}F\,e^{-i\omega t + ik\hat{z}} + \vec{e}_{12}F\,e^{-i\omega t - ik\hat{z}}\right) + \frac{1}{2}\left(\vec{e}_{32}F^*\,e^{i\omega t - ik\hat{z}} + \vec{e}_{12}F^*\,e^{i\omega t + ik\hat{z}}\right), \tag{8}$$

where $\vec{e}_{32}$ and $\vec{e}_{12}$ are the polarizations of the forward and backward laser fields. The dipole moment operator only affects the internal quantum numbers of the atom. In our system, the two laser fields are polarized in such a way that:

$$\langle 3|\hat{\mu}\cdot\vec{e}_{32}|2\rangle = \langle 1|\hat{\mu}\cdot\vec{e}_{12}|2\rangle \equiv \mu \tag{9}$$

are the only nonzero matrix elements.

To find the matrix expansion of the dipole interaction, we make use of the identity:

$$e^{+ik\hat{z}}|i,p\rangle = |i,p+\hbar k\rangle\ . \tag{10}$$

Equation 10 can be proved using the transformation between position and momentum space which can be found in many quantum mechanic textbooks: [3]

$$|i,p\rangle = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{+\infty} e^{i\frac{pz}{\hbar}}|i,z\rangle\,dz\ , \tag{11}$$

Operating on both sides of Equation 11 with $e^{+ik\hat{z}}$ yields:

$$e^{+ik\hat{z}}|i,p\rangle = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{+\infty} e^{i\frac{(p+\hbar kz)}{\hbar}}|i,z\rangle\,dz = |i,p+\hbar k\rangle, \tag{12}$$

Using the identity of Equation 10 and substituting Equation 8 into Equations 6 gives the matrix elements of the dipole interaction as:

$$\begin{aligned}
\hat{H}_{AL}|1,p\rangle &= -\hbar E e^{-i\omega t}|2,p-\hbar k\rangle - \hbar E^* e^{-i\omega t}\,|2,p+\hbar k\rangle \\
\hat{H}_{AL}|2,p\rangle &= -\hbar E e^{-i\omega t}|3,p+\hbar k\rangle - \hbar E^* e^{-i\omega t}\,|3,p-\hbar k\rangle \\
&\quad - \hbar E e^{-i\omega t}|1,p-\hbar k\rangle - \hbar E^* e^{-i\omega t}\,|1,p+\hbar k\rangle \\
\hat{H}_{AL}|3,p\rangle &= -\hbar E e^{-i\omega t}|2,p+\hbar k\rangle - \hbar E^* e^{-i\omega t}\,|2,p-\hbar k\rangle\ ,
\end{aligned} \tag{13}$$

where $E = \mu F/2\hbar$ is called the Rabi frequency of the laser field. [5]

Expanding the wave function in terms of the slowly varying dynamical variable $c_i(p,t)$ gives: