DESIGN OF A LOW COST GAMMA RAY SPECTROMETER TO INVESTIGATE SPECIAL RELATIVITY

James Bopp *
Division of Science (Physics)
Truman State University
Kirskville, MO 63501

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ABSTRACT

We designed a low cost NaI(Tl) scintillation detector and single channel analyzer to be used to measure the rest energy of the electron. We used the instrument to deduce the behavior of electrons following Compton scattering collisions in the crystal. We found a value of the rest energy of the electron to be (0.51±0.02) MeV and experimentally verified Einstein's energy-momentum relationship.

INTRODUCTION

The experimental investigation of special relativity in the undergraduate laboratory is often neglected due to the high cost and sophistication of instrumentation capable of measuring relativistic behavior. This means that many students feel that relativistic theory only applies to imaginary rockets traveling close to the speed of light and not to the real world. However, when studying the interaction between matter and gamma rays during events such as Compton scattering, one finds that relativistic behavior is the norm.

Gamma rays primarily interact with matter in three ways: a) via the photoelectric effect; b) via Compton scattering; and c) via pair production. The material we will be using in this experiment is a NaI(Tl) scintillation crystal, a material that converts the energy of an icident gamma ray into photons of visible light. In this paper, we will be concerned with the first two mechanisms.

Photoelectric Effect

The photoelectric effect is a phenomenon in which a photon (a gamma ray in this experiment) is absorbed by a tightly bound electron in the material. This electron is then ejected from the atom with an energy, E_e, equal to the difference between the photon energy, $E\gamma$, and the binding energy, E_B, of the electron to the atom.

$$E_e = E_\gamma - E_B \ . \tag{1}$$

In a NaI(Tl) scintillation crystal, all of E_e is deposited in the crystal's lattice. E_B shows up as a X-ray from the filing of the vacancy in the atom created by the ejection of the electron. This X-ray is also absorbed by the crystal. Therefore, nearly all of the energy of the gamma ray interacting with the crystal through the photoelectric effect is deposited in the detector.

Figure 1 shows an idealized output from a scintillation crystal that is being bombarded by single energy gamma rays. The photopeak shown is caused by the photoelectric absorption in the crystal. This peak corresponds directly to the energy of the incident photon. [1]

Compton Scattering

Compton scattering is essentially a sub atomic game of billiards. An incident gamma ray interacts with a free electron and the two scatter as depicted in Figure 2. The energy of the original gamma ray is divided between the energy of the scattered gamma ray and the recoil electron in a way that conserves energy and momentum. The energy of the scattered gamma ray is dependent on the scattering angle. The scattered photon frequently escapes from the scintillation crystal and only the energy of the recoil electron is absorbed by the crystal and reemitted as visible light.

Compton scattering in the crystal produces a continuous spectrum in the output as shown in Figure 1. The Compton Continuum shows electrons with kinetic energy beginning at almost zero, the photon just grazing the electron and scattering at a small angle, to some maxi-

James Bopp earned a B.Sc. in physics from Truman State University with minors in mathematics, philosophy and religion. The research that is discussed here was conducted during his last two years at Truman. It was sponsored by grants from the Univerisity. He is currently a Jesuit novice at St. Paul, MN. He hopes to study physics at the graduate level while he engages in the philosophical and theological studies necessary to become a Catholic priest.

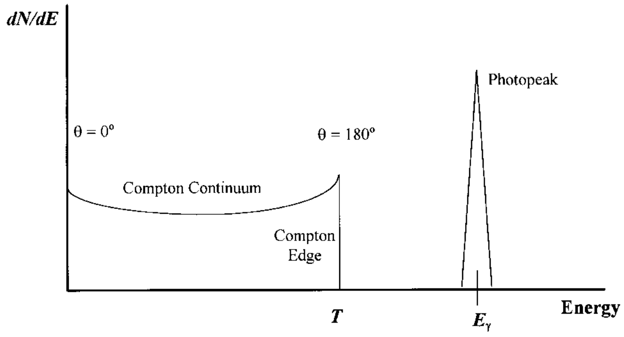

Figure 1
Idealized spectrum produced by a gamma ray stopping in a scintillating crystal. The photopeak corresponds to the energy of the incident gamma ray. The Compton edge corresponds to the maximum kinetic energy an electron can obtain through Compton scattering.

mum kinetic energy, T_{max}, known as the Compton Edge, where the photon back scatters at 180 degrees. [2] The energy corresponding to the Compton Edge can never equal the original energy of the photon because both energy and momentum are conserved in the interaction.

Relativistic Kinematics

Discussions on relativistic kinematics can be found in many textbooks. [3] Using the conservation of total energy and relativistic momentum, where:

$$E_e^2 = (cp)^2 + (m_oc^2)^2 = (T + m_oc^2)^2 , \tag{2}$$

where T is kinetic energy, the Compton edge energy, T_{max}, due to the scattering of a photon of energy $E\gamma$, is given by[4]:

$$T_{\max} = \frac{E_\gamma}{1 + 2\dfrac{E_\gamma}{m_oc^2}} . \tag{3}$$

where m_oc^2 is the rest energy of the electron. Solving Equation 3 for the rest energy gives:

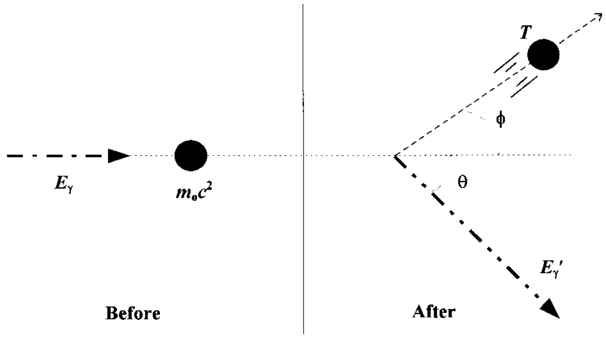

Figure 2
Schematic diagram of a Compton scattering event.

$$m_oc^2 = \frac{(cp)^2 - (T_{\max})^2}{2\,T_{\max}} = \frac{2\,E_\gamma\,(E_\gamma - T_{\max})}{T_{\max}} . \tag{4}$$

When one analyzes the scattering from a classical nonrelativistic standpoint, Equation 4 would take the form:

$$\left(m_oc^2\right)_{classical} = \frac{(cp)^2}{2\,T_{\max}} = \frac{\left(2\,E_\gamma - T_{\max}\right)^2}{2\,T_{\max}} . \tag{5}$$

Equations 4 and 5 only differ significantly when T_{max} becomes a significant fraction of the rest energy of the electron. Thus if one experimentally measures T_{max} and E_γ for gamma ray energies in the MeV region, one can experimentally verify the relativistic kinematics formulae.

THE APPARATUS

There is much information on how to apply gamma ray spectrometry to the study of special relativity. [5] The typical gamma ray spectrometer functions in the following way. A gamma ray enters the scintillating crystal where it interacts with the atoms in the crystal through the photoelectric effect or Compton scattering. In either case, the disturbance of the electrons in the crystal results in a flash of light whose intensity is directly proportional to the kinetic energy of the agitated electron. This flash of light is reflected into a photomultiplier tube (PMT) which uses photoelectric absorption and secondary emission to transform the light into an electrical pulse whose voltage magnitude is directly proportional to the intensity of the light, and therefore, proportional to the energy deposited by the gamma ray in the crystal.

What is unique about what we have done is the cost and simplicity of the apparatus. Traditional apparatus for gamma ray spectroscopy in the undergraduate laboratory consists of a scintillation detector (a scintillating crystal attached to a photomultiplier tube) which sends signals to a computer based multichannel analyzer. Others use high purity germanium detectors that must be stored and operated at cryogenic temperatures. In general, these systems cost thousands of dollars.

Our spectrometer is certainly more primitive than the commercially available models, but it is capable of obtaining comparable results for less than a thousand

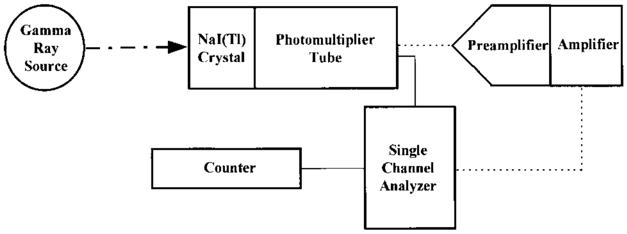

Figure 3
Schematic diagram of the electronics used to construct the Gamma Ray Spectrometer used in this experiment.

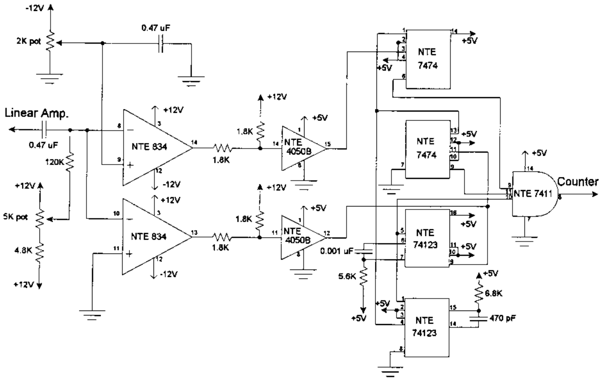

Figure 4
Schematic diagram of the Single Channel Analyzer

dollar investment. We purchased a 2” x 2” NaI(Tl) crystal and a 10 stage box and grid PMT [6] and a high voltage power supply to operate the PMT [7]. We built our own voltage divider for the PMT. The detector we constructed produced large signals (around 5 V for a 0.662 MeV gamma ray), so no preamplifier was necessary to analyze the signal. To analyze the various voltage pulses produced by the PMT, we built a single channel analyzer (SCA) out of five common integrated circuits and a few peripheral components. [8] Figure 4 is a schematic diagram of the SCA.

The SCA consists of a small fixed voltage window which is systematically swept through the full range of the possible voltages input to the device. Any pulse which falls above or below the window is blocked, while pulses whose peaks fall within the window produce a logic pulse which is sent to a counter. (see Figure 5).

The relative number of counts per time are plotted as a function of the voltage value of the center of the window. This histogram is called the gamma ray spectrum. Figure 5 shows a spectrum we obtained from a ^{137}Cs source. It should be compared to the ‘ideal’ spectrum shown in Figure 1. The SCA is calibrated by producing gamma ray spectra for several sources that have known energy photopeaks and determining the energy value corresponding to any given window value (see Figure 6).

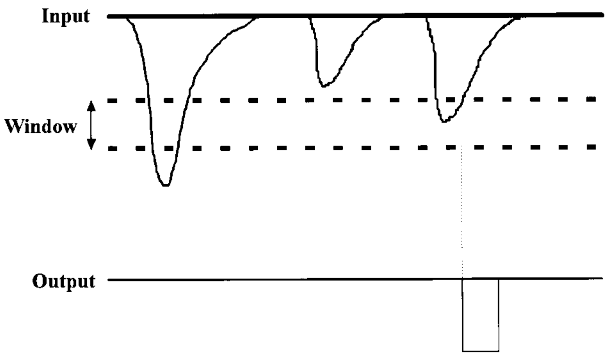

Figure 5
Basic operation of a single channel analyzer. Only those pulses whose peak value falls inside the window register cause a logic pulse to be sent to the counter.

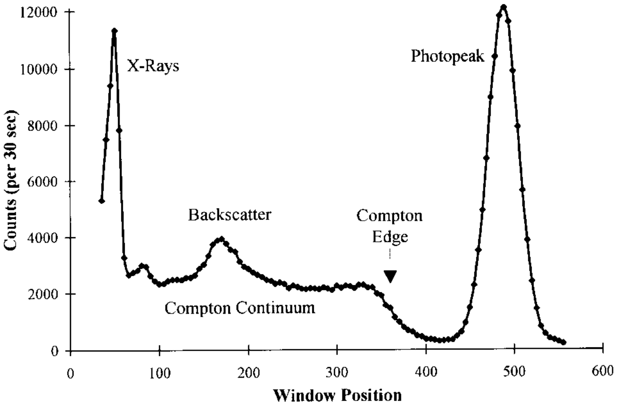

Figure 6
Experimental gamma ray spectrum for a 137Cs source. Compare this to Figure 1.

The total cost of our spectrometer was less than one thousand dollars. Our spectrometer has a resolution (FWHM, ^{137}Cs) of 7.7%, which is typical of commercial NaI(Tl) gamma ray spectrometers. The disadvantage of using an SCA is that it takes over an hour to collect the spectrum data using it.

RESULTS USING OUR SPECTROMETER

We carefully measured the gamma ray spectrum for 8 different sources. From these data, we determined values of the Compton edge (T_{max}) and the energy of the gamma ray ($E\gamma$). Table 1 shows these results.

Figure 7 is a plot of the rest energy vs T_{max} using Equation 5, the relativistically correct analysis of Compton Scattering. The zero slope fit to the data in Figure 8 shows an invariant rest mass with a value (0.51±0.02) MeV. This value overlaps the currently accepted value of the rest energy of the electron.

Figure 8 shows a plot of the momentum of the electron that is back scattered as a function of the measured kinetic energy (T_{max}). The solid line is what one would expect using the classical approximation. The dashed line in Figure 8 is the relativistically correct theoretical relationship between momentum and kinetic energy.

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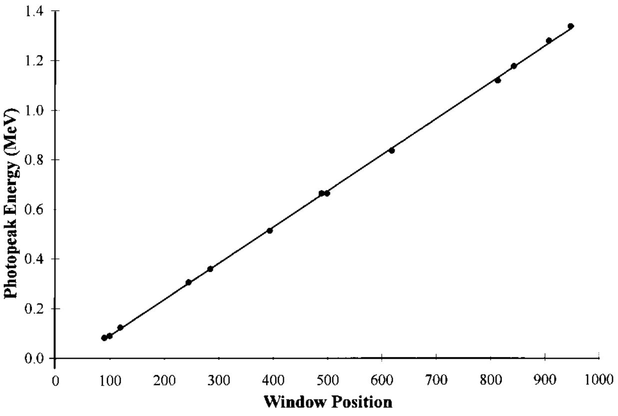

Figure 6
Calibration graph created by plotting known photopeak energies against measured photopeak positions. The line is a best fit using linear regression. The error bars are smaller than the data points.

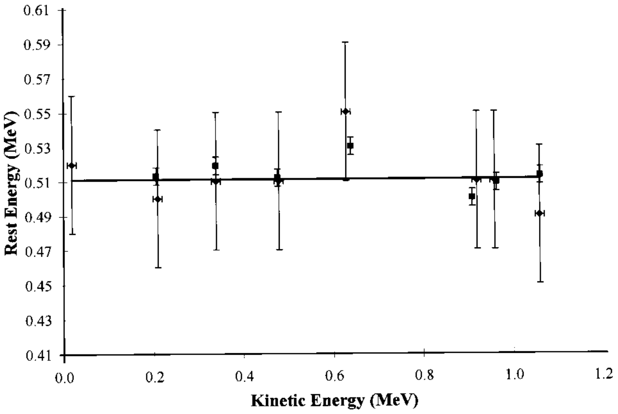

Figure 7
Relativistic calculations for the electron's rest energy plotted against its kinetic energy. The solid line is at 0.511 MeV. The square data points are from measurement taken with an HPGe detector and are shown for the sake of comparison.

allowing the use of his HPGe detector data.

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Professor Kenneth Hahn
Division of Science
Truman State University
Kirksville, MO 63501-4221

Source	Experimental Photopeak Energy (MeV)	Accepted Photopeak Energy (MeV)	Experimental Compton Energy (MeV)
^{22}Na	1.265±.007	1.2745	1.06±.01
^{22}Na	0.511±.007	0.5110	0.34±.01
^{60}Co	1.171±.007	1.1732	0.96±.01
^{65}Zn	1.127±.007	1.1155	0.92±.01
^{54}Mn	0.838±.007	0.8340	0.63±.01
^{137}Cs	0.664±.007	0.6616	0.48±.01
^{133}Ba	0.358±.007	0.3560	0.21±.01
^{109}Cd	*0.083±.007*	*0.0880*	*0.02±.01*

Table 1
Measured and accepted values of the photopeak and our measured values for the Compton edge energies.

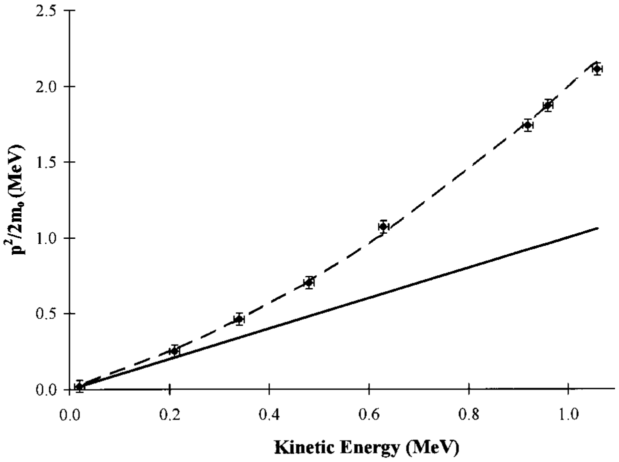

Figure 8
The classical and relativistic relationships between energy and momentum. The solid line represent the classical predictions, the dashed line the relativistic predictions.