EXPLORATION IN TEN PIN: A SIMPLIFIED MODEL OF A BOWLING SYSTEM

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ABSTRACT

The complicated physical system of bowling is simplified to a two-dimensional model, where the pins and ball are represented by hard disks, friction is neglected and straight line motion and elastic collisions are assumed. We show that, while not entirely realistic, the model reflects some of the behavior we see occurring in a real, three-dimensional bowling system. This simplified model reveals tremendous sensitivity to the initial parameters of the system and provides insight into the nature and dependence of the elusive strike zone.

INTRODUCTION

The game of bowling uses a complicated physical system. If all of the factors that affect the system in reality were taken into consideration, a mathematical representation of the system would be nearly impossible to develop. The list of factors that realistically complicate the system is large: spin on the ball, the toppling and rolling effect of the oddly shaped pins, the inelastic nature of collisions... [1] Countless bowlers, from novice to professional, have at some point in their careers wondered, when it seems as though the ball travels repeatedly with the same speed along the same path, why that sometimes a strike occurs and sometimes 2 or 3 pins are left standing. The large number of physical factors that complicate the system seem to be the cause of the inherent unpredictably and seeming irreproducibility of the bowling system. In this paper, we will strip away many of the complicating factors and examine the bowling system in an idealized form.

Jocelyn is a senior math major at Carleton College. This research was done in the summer of her sophomore year, when she interned with the amiable and supportive Carleton College physics department. She is currently considering graduate schools in architecture and theater, although she loves writing and may pursue a career in journalism. When she does get a job, she hopes she can ride her bike to work. Jocelyn has a passion for Shakespeare, and is often distracted from problem sets by daydreams of playing Juliet, Lady Anne, Ophelia or Lady MacBeth.

There are a number of fixed parameters in the bowling game. These come directly from standards set by the American Bowling Congress (ABC).[2] The pins are arranged in the shape of an equilateral triangle and are numbered from 1 to 10. By convention, the head pin is numbered 1. The numbering continues back row by row, increasing in value from left to right. The pins are 1 foot apart, measured from center to center. The largest circumference of the pin is 0.20 feet (or 2.5 inches). The bowling ball has a radius of 0.36 feet (4.3 inches) and can weight from 9 pounds to 16 pounds. According to ABC regulations, the pins must weigh between 2 pounds 14 ounces and 3 pounds 4 ounces. The alley is 3.5 feet across. The distance from the center of the head pin to the back pit is 3.0 feet.

THE MODEL

Assumptions

We assume a frictionless level plane as the alley. The ball and pins are represented by two-dimensional disks. Collisions are assumed to be completely elastic. Between collisions, the disks move in straight lines. These assumptions mean that in our model, unlike in a real bowling system, the outcome is independent of the initial speed of the ball. In spite of this feature, the essence of the bowing system is preserved.

In our simulation, we only consider the cases where initially all 10 pins are standing and where the first collision occurs between the ball and the head pin. Considering other situations would illuminate the other bowling phenomena, but our interest lies in exploring the nature of strikes (where all 10 pins fall).

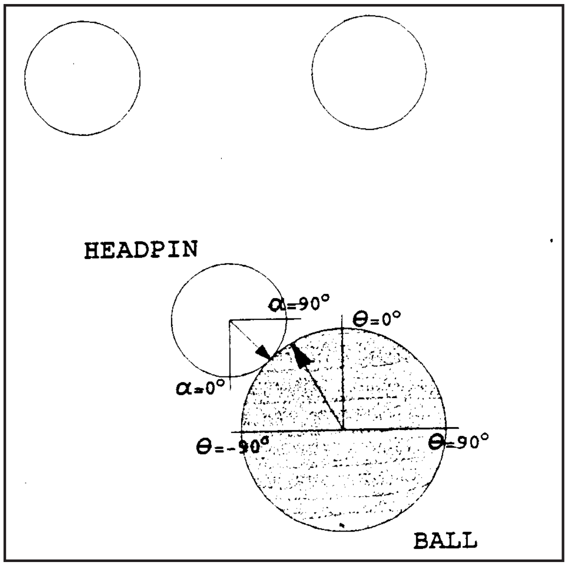

Figure 1
Conventions used to describe the ball's initial position, α, and the angle of the initial velocity of the ball, θ. There the ball is at position $\alpha = 45^\circ$ with a velocity represented by the arrow in the ball, $\theta = 30^\circ$

Parameters

The relevant parameters fall into two categories: those that have fixed values and those that can be varied. The fixed parameters are the physical arrangement of the pins, the radii of the pins, the radius of the ball and the location of the edges of the alley (all specified by the ABC). The parameters that can be varied are the initial point of contact between the ball and the head pin, the angle of the initial velocity of the ball and the ball to pin mass ratio.

The dimensions of the alley are used as boundary lines in our system. When a disk goes beyond these lines, it no longer participates in the collision action. In reality, a pin has the opportunity to bounce off the sides of the alley and re-enter the system, but in our model, this is not allowed.

We define the variable parameters in terms of two distinct Cartesian axes, imposed on the ball and on the arrangement of the pins as shown in Figure 1. The initial contact point is determined by the angle, α, of the line connecting the contact point and the center of the head-pin, shown in Figure 1 as the arrow pointing towards the ball. The point closest to the bowler on the circumference of the head pin corresponds to $\alpha = 0^\circ$. The direction of the initial velocity of the ball is the angle θ. When the velocity of ball has no sideways component, we say that $\theta = 0^\circ$. Values of α range between 0° and 90°, values of θ have a range of 180° perpendicular to the axis of the collision (which turns out be dependent on α). This range includes all possible realistic values of θ and extends to include values that are less possible in a real bowling setting. (For example, some of the combinations have a negative component in the direction of the bowler: back up the bowling alley. Professional bowlers put quite a bit of spin on the ball, which allows for fairly large angles of initial velocity, but it never turns so much that the ball is heading back towards the bowler.)

The final variable parameter is the ball to pin mass ratio, which we call μ. Based on ABC regulations, μ can range from 2.5 to 5.5.

The Simulation

We programmed the simulation in MATHEMATICA™. The algorithm consists of a few steps that are cycled through a number of times until a final condition is met. The initial sate of the system (the program's input) is a list of the position and velocity of the ball and each pin. The ball's velocity is determined by θ and its position by α. The pins are all at rest and set up in their standard triangular formation.

The first step the simulation performs is to look at the initial state and calculate the new velocities of the disks that have collided. This is done simply using the laws of conservation of energy and conservation of momentum, plugging in the mass of the colliding objects as well as their initial positions and velocities. The calculations give the post collision velocities of the objects. This is the post-collision state. The new values are then substituted into the input state in the place of the old velocities. From the positions and velocities of all the pieces, the program calculates for each pair the time it would take for the two particles to collide (if they collide at all). From the list of collision times, the one with the lowest value is selected. The simulation does not allow for the possibility that two collisions could occur simultaneously, because except for the case of a directly centered collision, this would be a highly unlikely occurrence. Because of this feature of the program, the outcome when $\alpha = \theta = 0^\circ$ is disregarded.

Once the program has determined the least collision time, it calculates the position of each particle and replaces the old values with new ones. This is the pre-collision state.

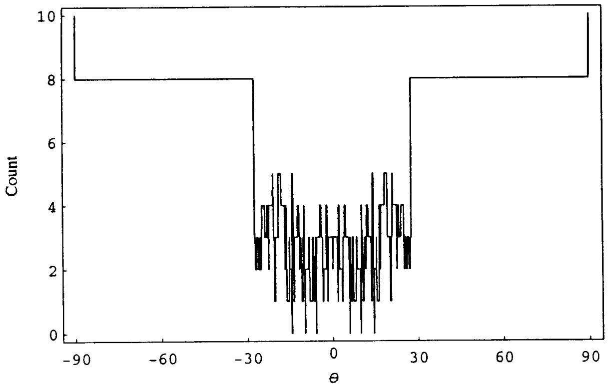

Figure 2
Number of pins remaining vs. the angle of velocity at the collision, θ. Here $\alpha = 0^\circ$ and θ varies by 0.1˚.

The pre-collision state then becomes the initial state and the program repeats itself. When the velocities of all the particles are such that no further collisions occur, the program ends, reporting the position and velocity of each particle just after the last collision. This is the final state, which indicates which pins remain standing.

The simulation produces raw data, a list of the position and velocity of each particle after each collision in the system. The first element of the list is the initial state, the last element is the final state. From this information, we can create a static plot, showing the paths of all the particles together without regard to time. The raw data also can be simplified by counting the pins that were never hit. This is the number of pins left standing, and is called the count.

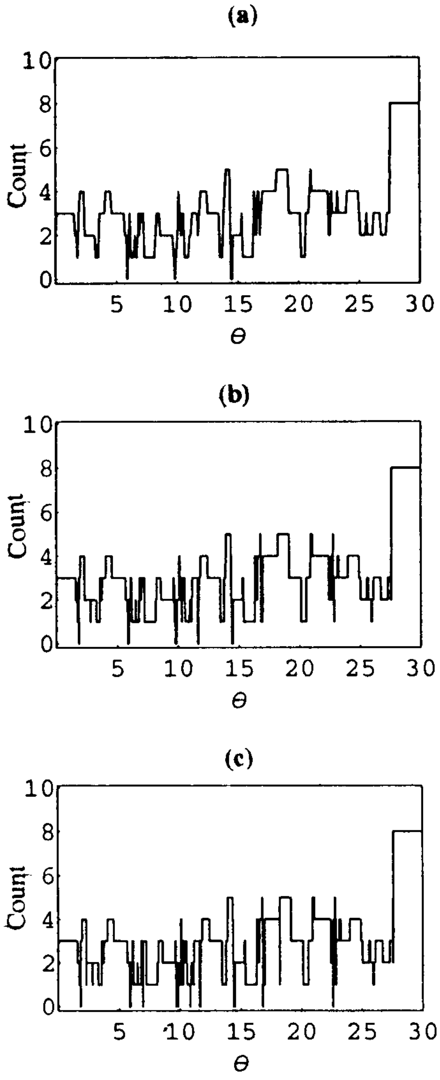

Figure 3

Number of pins remaining vs the angle of initial velocity, q. The count is calculated for θ varying by a) 0.1˚, b) 0.01˚ and c) 0.005˚. For each graph α = 0˚ and μ = 4.

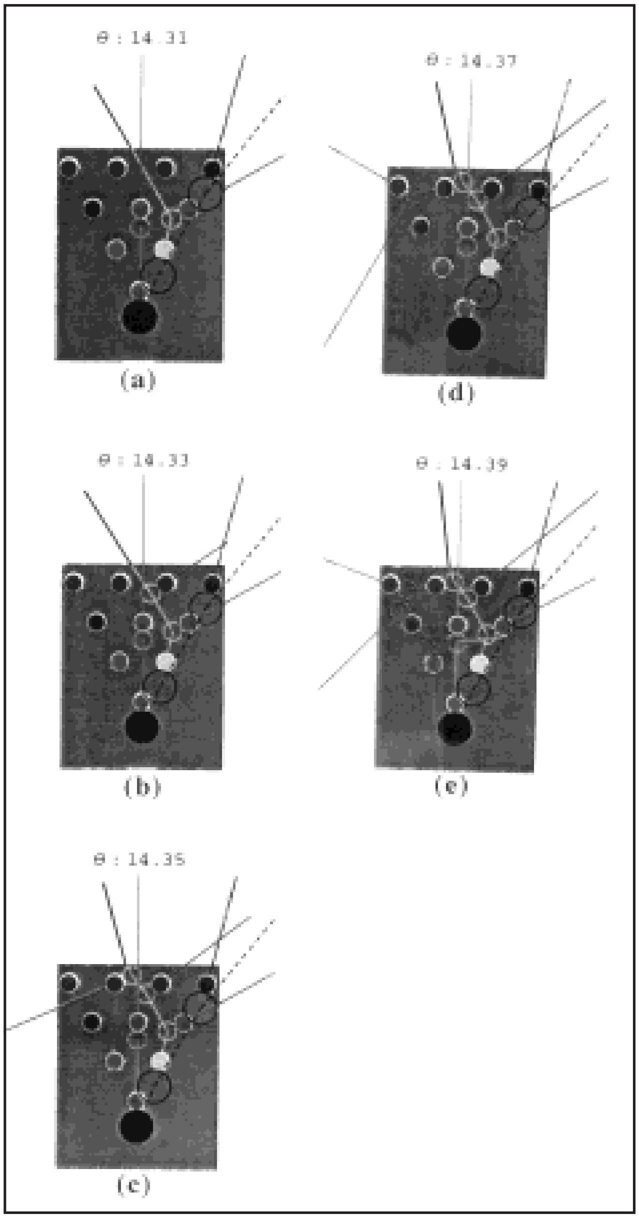

Figure 4

Static plots of the paths of all of the disks for distinct values of θ. Time is not represented in these graphs.

CALCULATION AND DISCUSSION

Sensitivity of the System

<u>Varying the initial velocity</u>

We fixed the initial point of contact between the ball and the head pin to at $\alpha = 0^\circ$, and the mass ratio to at $\mu = 4$ (a moderate mass ratio), and varied the angle of the initial velocity of the ball from $\theta = -90^\circ$ to $+ 90^\circ$ in discrete increments.

Figure 2 shows the count (number of pins remaining) as θ varies in increments of 0.1 degrees. The graph appears symmetric, which gives us some confidence in the program. The graph shows sharp transitions and a large

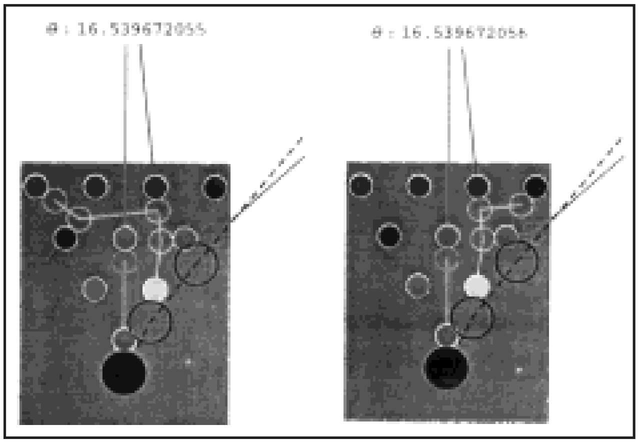

Figure 5
Static plots of the paths of all of the disks for two distinct values of θ which differ by a billionth of a degree.

fluctuation in the number of pins left, even when θ varied only by a fraction of a degree. The wildly fluctuating outcomes raise questions about the behavior of the system. The interesting behavior seems to occur at values of |θ|<30˚. Since Figure 2 is symmetric, we restrict our study to θ between 0˚ and 30˚.

Figure 3 shows the results of the simulation with increments of θ of: a) 0.1 degree, b) 0.01 degree and c) 0.005 degrees. Figure 3 shows that increasing the number of data points sampled reveals fluctuation in the system not captured in Figure 2. In Figure 3a, there are only 3 strikes, in Figure 3b there are 5 strikes and in Figure 3c there are ten strikes. This illustrates the sensitive nature of the system to initial conditions.

To understand the interactions that determine such sensitive behavior, we examine in detail the path of the particles where the dramatic fluctuations occur. Figure 4 shows static plots of the paths of all the particle as θ changes from 14.31˚ to 14.39˚ in increments of 0.02˚. The change in θ seems insignificant, but variations in the behavior of the whole system that result from such small changes in initial conditions are quite apparent. Small changes in the velocity of the ball create more visible changes in the velocity of the 3-pin. These changes make the difference between whether the 3-pin hits just one pin (Figure 4a) or several pins. The same things happens with the pins that the 3-pin hits, they either miss the few pins that remain standing (Figure 4d) or have just the correct velocity to hit them, (Figure 4e).

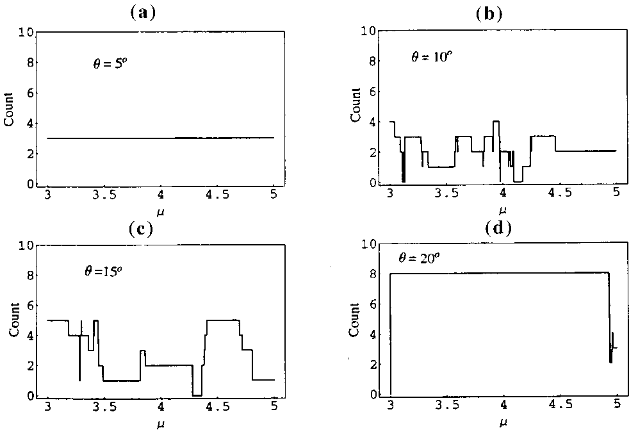

Figure 6
Number of pins remaining vs the mass ratio, μ, for different values of the initial velocity of the ball, θ. In each graph, α = 0˚

The manner in which small changes in the initial velocity of the ball affects the behavior of the entire system is illustrated in Figure 5, which shows the resulting paths of the particles as θ is changed by a billionth of a degree. In Figure 5a, the 3-pin hits the 9-pin and heads off in the negative direction (to the left) . In Figure 5b, the 3-pin hits the 9-pin and heads off in the positive direction. In Figure 5a, just before the 3-pin hits the 9-pin, it has a minute negative velocity component (relative to the axis of collision), while in Figure 5b, it has a minute positive position component. There is a single point where the velocity changes from having a negative component to having a positive component, which accounts for the sharp transition in the direction of the velocity of the 3-pin.

<u>*Varying the initial point of collision*</u>

The same sort of behavior as discussed in the previous section occurs when θ and μ are kept constant and α is varied. The count seems to depend as sensitively on α as it does on θ.

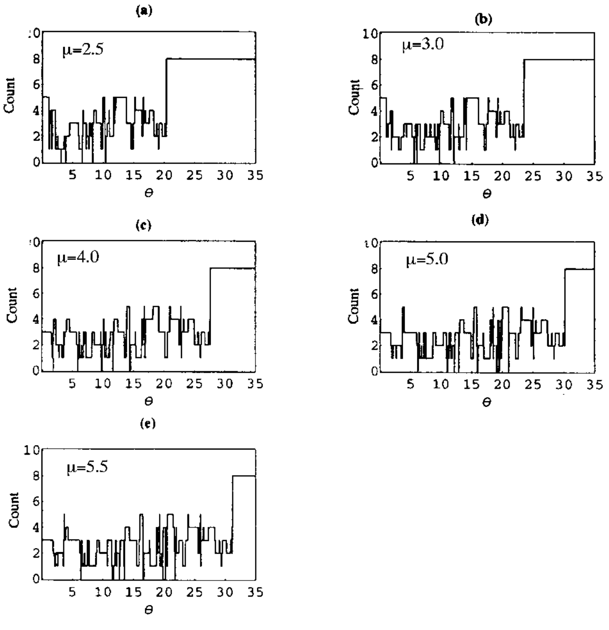

Figure 7
Number of pins remaining vs the angle of the velocity of the ball, θ for different values of the mass ratio, μ. In each graph, α = 0˚.

Varying the mass ratio

Variations in μ have a less dramatic influence on the system than variations in α and θ. Figure 6 shows the effects of varying the ball to pin mass ratio for 4 different values of θ. In each case, $\alpha = 0^\circ$. While there is some variation in the outcomes, the fluctuations are not as wild as those seen in the previous results. The results shown in Figure 6a and Figure 6c show that for certain initial angles, the count stays essentially constant as the ball to mass ratio changes.

Professional bowlers typically bowl with a 16 pound ball, the heaviest that the ABC allows. Figure 7 shows the number of pins that remain for $\alpha = 0^\circ$, θ varying as in Figure 2, for 5 different values of μ ranging from 2.5 to 5.5. The structures of each of these graphs are very similar; for larger values of θ, the pattern formed by the fluctuations is the same. There seems to be a point where the count jumps to 8 and remains there for a large range of θ. The value of θ at which this occurs increases with μ. As μ increases, the window of angles within which more than two pins will be knocked down widens. This means that using a heavier ball improves the bowler's change of knocking down at least 2 pins.

The similarity in structure of the 5 sections of Figure 7 explains why when we vary μ as in Figure 6, for some θ values, we see wild fluctuations in the count and for others, the value is constant. On each graph in Figure 7, there are some places where a particular number of pins remain over a significant range of angles, creating a flat line on the graph. These flat ranges would then overlap in graphs of the type in Figure 6, for certain values of θ.

The Quest for the Strike Pocket

Bowlers are perpetually seeking the elusive 'pocket', the small range between the head pin and the 3-pin (for right handers) or between the head pin and the 2-pin (for left handers) that if hit yields strikes. In our model, this pocket is dependent on a combination of the angles α and θ.

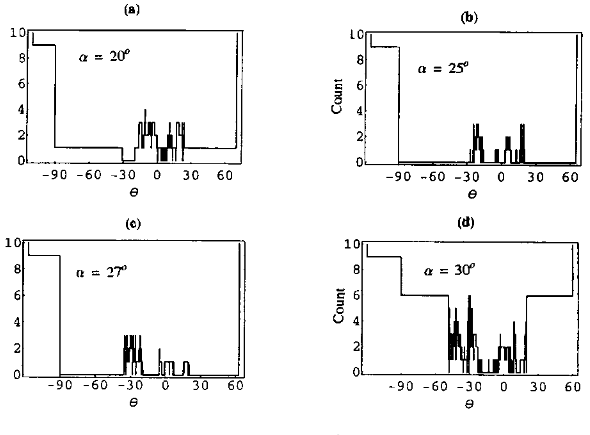

Figure 8
Number of pins remaining vs. the angle of the velocity, θ, for different values of the initial position of the ball, α. For each graph, $\mu = 4$.

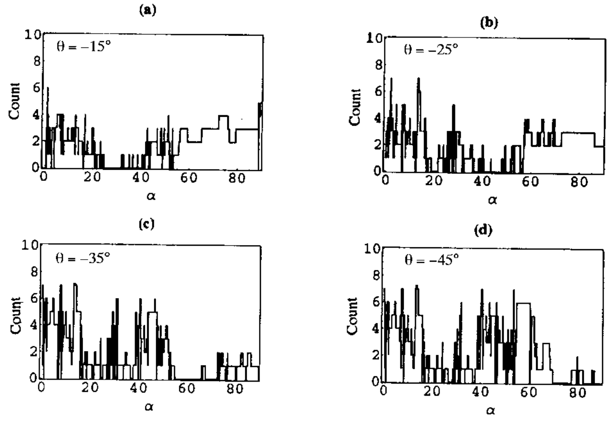

Figure 9
Number of ins remaining vs the initial position of the ball, α for different values of the angle of the initial velocity of the ball, θ. For each graph, $\mu = 4$.

We search for this 'pocket' by holding α constant and varying θ. Recall that Figure 2 shows the effects of varying θ at the fixed position $\alpha = 0$. Figure 8 shows the effect of varying θ through 180° for several different values of α. These figures show that although the possibility for getting a strike exists at many values of α, at certain initial contact points, there are larger ranges of θ which will yield strikes. In Figure 2, there are only a few isolated values of θ that result in a strike (count of zero). In contrast, Figure 8a shows a count of zero for the range of θ values from about -30° to -20°. This is the first suggestion of what might be called a 'strike zone'. In Figure 8b, we see much more impressive results. At a position of $\alpha = 25^\circ$, any θ value in the range from -30° to $+30^\circ$ will yield a strike. Figure 8c , where $\alpha = 27^\circ$, shows much the same effect. At a position of $\alpha = 30^\circ$, as illustrated in Figure 8d, the chance of getting a strike again becomes less probable. Even though at $\alpha = 25^\circ$ and $\alpha = 27^\circ$ there are large ranges of θ at which strikes occur, there are variations within those ranges where some pins are left. These data show that the outcome of the bowling system is sensitively dependent on both the initial velocity of the ball and the location of the initial point of contact.

The 'pocket' with regard to the initial point of contact

In contrast to Figure 8 where α was held constant and θ varied, Figure 9 shows the effect of varying α from 0° to 90° for several different fixed values of θ. In Figure 9 it is much more difficult to pinpoint a significant range of α that consistently yields strikes. There appears in Figure 9 a trend in the values of α that correspond to the most strikes. In Figure 9a, $\theta = -15^\circ$, the most concentrated range of strikes occurs in the range $\alpha = 20^\circ$ to $\alpha = 40^\circ$. In Figure 9b, where $\theta = -25^\circ$, the concentrated ranges of strikes has moved to the right, α values as high as 55° yield mostly strikes. The effect continues in Figures 9c

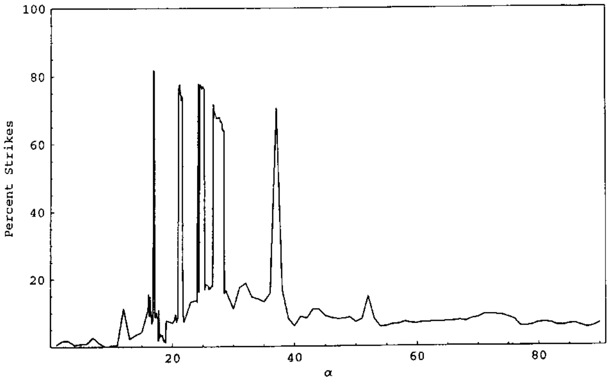

Figure 10
The percentage of time a strike occurs vs the initial position of the ball, α. For each value of α, the angle of the initial velocity, θ, ranges 180˚ in increments of 0.01˚. Throughout, μ = 4.

and 9d. There is a gradual increase in the values of α that yield strikes as θ becomes more negative.

The nature of the strike zone

The data suggest that getting strikes is dependent on both the angle of the velocity of the ball and the initial collision point between the ball and the head pin. Figure 10 shows a combination of all the data. For each value of α, θ ranges 180˚ in increments of 0.1˚. The total number of times the combination of a particular α and all increments of θ yielded a strike was calculated. Figure 10 presents the data as the percentage of times a strike occurs for each different initial contact point.

There are almost no values of α for which the percentage of time a strike occurs is zero. This means that it is possible to get a strike at virtually every value of α. However, the spikes that occur between α = 15˚ and α = 40˚ indicate that collision points for which the majority of the θ values would yield a strike. The values of α to which those peaks correspond are the points of collision toward which bowlers strive, those scattered thin spikes seen in Figure 10 make up the strike zone.

Figure 8 shows that there is no solid, thick range of a values for which the majority of θ values yield strikes, nor is there a single value of α for which 100% of the values of θ yields strikes. The opportunity exists to get strikes, but the position at which they are most likely to occur are thin ranges at which even 20-30% of the total values of the initial velocity angle do not yield strikes. The 'strike zone' that we have found is not a solid continuous range as the name might suggest, but a patch work of combined position and velocity values.

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