DETERMINATION OF THE TEMPERATURE DEPENDENCE OF YOUNG'S MODULUS FOR STAINLESS STEEL USING A TUNING FORK

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ABSTRACT
Tuning forks have been used as high quality frequency standards for decades. Musicians are well aware of the effect that temperature has on the tuning of their instruments. Those effects are generally attributed to either a change in the speed of sound in air (for wind instruments) or thermal expansion for string and percussion instruments. We show that in the case of a tuning fork, thermal expansion is a minor consideration. The primary source of the temperature dependence of the tuning fork is caused by a temperature dependence in the Young's modulus of the material out of which it is made. The stiffness of the fork changes slightly with temperature, causing a change in the resonant frequency of the tuning fork. We use this effect to determine an empirical formula for the temperature dependence of stainless steel.

INTRODUCTION

The first person to tune musical instruments with a tuning fork was Pythagoras in the sixth century BC. It is believed that he created the diatonic musical scale. [1] Most applications of tuning forks use the fork as a frequency reference standard. Musicians usually tap a tuning fork to get a fixed pitch, against which they tune their instruments.

At one time, physicists used tuning forks as frequency standards. Now, the physics of tuning forks is interesting in other ways. Research is done on the modes of vibration or the nonlinear motion of the tines. [2] The literature, however, does not have much to say about the temperature dependence of tuning forks. [3,4]

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Musical instruments have pitches that vary with temperature. The variation of pitch for percussion and string instruments is generally attributed to the thermal expansion of the instrument. For wind instruments, variations in the velocity of sound in air cause the frequency variation. The variation in pitch is very noticeable in wind instruments of the flute type. Consequently, wind instruments must be 'warmed up' before tuning; the player's breath will determine the actual temperature within the instruments. Before air conditioning for auditoriums, orchestras would adjust the pitch of their instruments to match the piano, because the pitch of the piano does not have much temperature dependence. Even so, the piano itself must be tuned at some temperature.

In the mid 19th century, there was a discussion about the appropriate reference temperature at which an instrument was tuned. In 1859, the French Commission chose 50 F as the standard pitch temperature. Later on, the British Standard Institution recommended 20 C or 68 F. The standard pitch and its associated temperature for instruments today is 440 Hz at 20 C. [5]

Literature in the area of tuning fork acoustic behavior is somewhat limited. Lord Rayleigh did some research concerning tuning forks; he observed that small masses

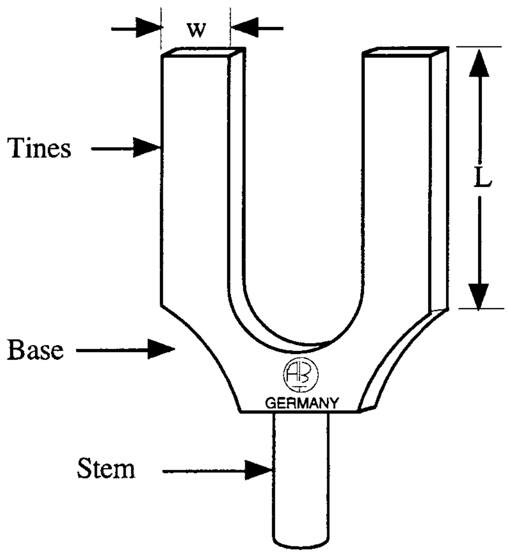

Figure 1
Schematic diagram of the tuning fork showing the relevant dimensions. The trademark etched on the base was not identified in any of our database searches.

hanging from the tines decreased fundamental stem motion. [6] Surprisingly little information on the temperature dependence of the tuning fork resonance frequency can be found. A previous experiment done by University of Wisconsin - River Falls students found that the resonant frequency decreased as a linear function of temperature in the temperature range between 24 C and 30 C. [7]

Thermal expansion, which is generally cited as the cause of the shift in frequency for percussion and string instruments, cannot account for the shift of the resonant frequency. The temperature dependence of Young's modulus has been demonstrated to be the dominant effect in determining the temperature dependence of the resonant frequency of small tuning forks made of materials such as crystalline silicon and zirconium. [8] In this paper, we extend this research to show how the temperature dependence of Young's modulus affects ordinary tuning forks.

THEORY

A simple tuning fork, such as the one shown in Figure 1, can be thought of as two straight bars, each of which is clamped at one end. From an examination of how the bars bend when they vibrate, the resonant frequency, f_n, can be shown to be: [8]

$$f_n = \frac{\pi}{16\sqrt{3}} \frac{w}{L^2} \sqrt{\frac{E}{\rho}}\, c_n , \qquad (1)$$

where w and L are the width and length of the tuning fork tines; E is Young's modulus; ρ the density of the material; n, is the mode number, an integer greater than 0; and c_n is a multiplication factor that depends on the mode. c_n is represented by the sequence:

$$c_n = (1.194)^2, (2.988)^2, (5)^2, \ldots ,(2n-1)^2 \quad n > 2. \qquad (2)$$

The temperature dependence of the expansion of the bars can be modeled as:

$$\begin{aligned} w(T) &= w_o\left[1 + \int_{T_o}^{T} \alpha_l(T)\, dT\right] \\ L(T) &= L_o\left[1 + \int_{T_o}^{T} \alpha_l(T)\, dT\right] \\ \rho(T) &= \frac{\rho_o}{\left[1 + \int_{T_o}^{T} \alpha_v(T)\, dT\right]} , \end{aligned} \qquad (3)$$

where w_o, L_o and ρ_o are the dimensions and density at temperature T_o, α_l is the linear coefficient of thermal expansion, and $\alpha_v \approx 3\alpha_l$ is the volume coefficient of thermal expansion.

In the small temperature range of this experiment, the coefficients of thermal expansion are approximately constant, so Equations 3 can be written as:

$$\begin{aligned} w(T) &= w_o\left[1 + \alpha_l(T - T_o)\right] \\ L(T) &= L_o\left[1 + \alpha_l(T - T_o)\right] \\ \rho(T) &= \frac{\rho_o}{\left[1 + \alpha_v(T - T_o)\right]} . \end{aligned} \qquad (4)$$

When Equation 4 is inserted into Equation 1, the temperature dependence of the frequency, $f_n(T)$, becomes:

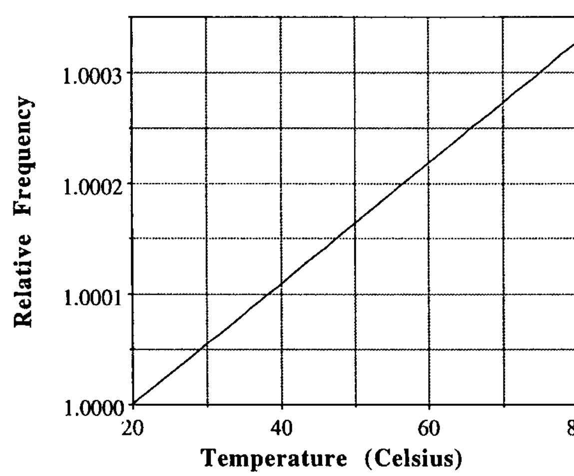

Figure 2
A numerical prediction of the effects of thermal expansion on the resonant frequency, ignoring any possible temperature dependence of Young's modulus. Note the very modest increase in frequency.

$$\frac{f_n(T)}{f_{n,o}} = \frac{\sqrt{\left[1 + \alpha_v(T - T_o)\right]}}{\left[1 + \alpha_l(T - T_o)\right]}, \qquad (5)$$

where $f_{n,o}$ is the frequency at temperature T_o. Figure 2 is a plot of the relative frequency over a temperature range used in this experiment for our tuning forks. The slope of the line is positive, with a value of 5 parts/million/C which is essentially zero. The actual behavior of the tuning fork shows a decrease in frequency of 1 percent as the temperature increases in this range. Hence a simple thermal expansion model does not predict the correct temperature dependence.

In the derivation of Equation 5, we assumed that Young's modulus is independent of temperature. Including a possible temperature dependence for Young's modulus gives:

$$\frac{f_n(T)}{f_{n,o}} = \frac{\sqrt{\left[1 + \alpha_v(T - T_o)\right]}}{\left[1 + \alpha_l(T - T_o)\right]} \sqrt{\frac{E(T)}{E_o}}, \qquad (6)$$

where $E(T)$ is Young's modulus at temperature T and E_o is Young's modulus at temperature T_o. The results shown in Figure 2 imply that we can ignore the thermal expansion terms. Rewriting Equation 6 gives:

$$\frac{E(T)}{E_o} = \left(\frac{f_n(T)}{f_{n,o}}\right)^2 \frac{\left[1 + \alpha_l(T - T_o)\right]^2}{\left[1 + \alpha_v(T - T_o)\right]}. \qquad (7)$$

We use Equation 7 to transform our values of relative frequency into relative Young's modulus.

THE EXPERIMENT

Alloy determination

The tuning fork we used was made of an unknown composition steel. A search of physics catalogs and a computerized trademark (see Figure 1) search of registered companies as well as discussions with the music department were fruitless in determining the composition of the steel used in the manufacture of the forks. So, we decided to determine the particular stainless steel by determining its density.

To determine the density of the tuning fork, we employed a technique used by geologists to find the density of irregular objects known as a Jolly balance. We measured the weight of the tuning fork in air, W_{air}, and in distilled water, W_{water}. The density is found as:

$$\rho = \rho_{water} \frac{W_{air} - W_{water}}{W_{air}} = 7.767 \pm 0.006\, g/cm^3. \qquad (8)$$

where $\rho_{water} = 0.998203$ g/cm^3. We compared this result with density information on various types of stainless steel. [10] We concluded that the tuning fork was most probably formed from a cutlery stainless steel. The cutlery stainless steel included 0.2% to 0.4% C, 0.2% Si, 0.35%Mn and 12% to 15% Cr; the remaining content is Fe.

Temperature determination

The temperature of the tuning fork was determined by an oven controlled by a variable auto transformer. Thermistors [11] were used to monitor the temperature. One was placed on a dab of glycerin on the metal plate clamping the tuning fork in place. A second thermistor was suspended in the air of the oven to measure the air temperature. The calibration data supplied by the manufacturer were fit to an appropriate series and used to convert the thermistor resistance, measured by a multimeter [12], in kΩ into temperature values in C. The uncertainty in our temperature values was ±0.1 C.

We monitored both the air temperature and the tuning fork temperature to avoid thermal lag. We found that the best way to minimize the thermal lag was to heat the oven and to regulate its rate of cooling by gradually decreasing the voltage supply. Using this technique, the two temperatures remained within 2 C of each other.

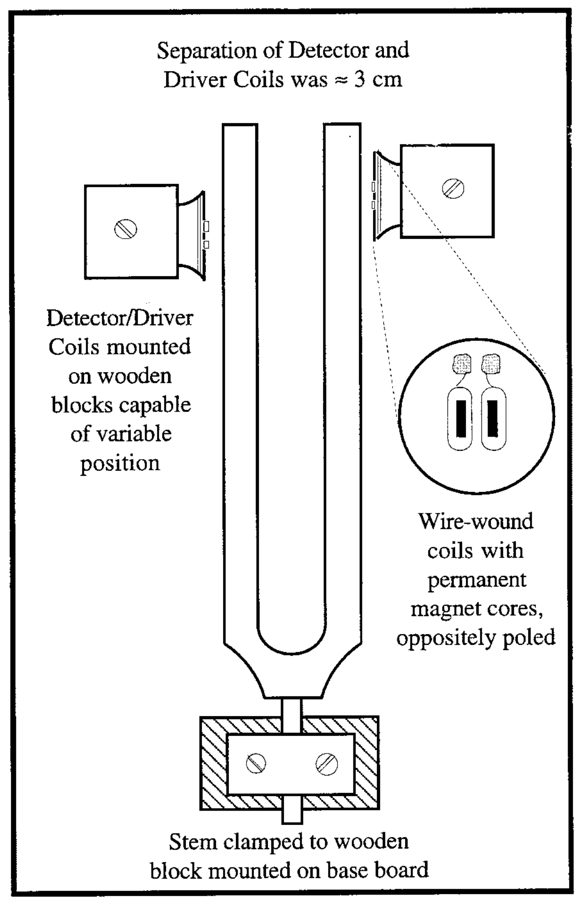

Figure 3

Schematic diagram of the apparatus. All components were mounted on a wooden base such that the relative positions of the coils and the tuning fork could be controlled. The inset shows an end-on view of one of the driver/detector coils.

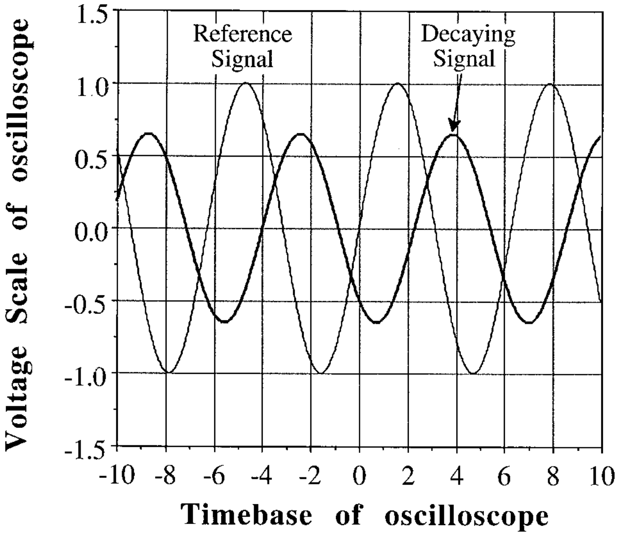

Figure 4
Representation of the oscilloscope trace. The signal generator provided a stable reference signal. As the signal from the tuning fork decays, any mismatch in frequency will result in a lateral shift of the decaying signal.

Driving the tuning fork
We used electromagnets salvaged from a pair of old earphones to drive and record the motion of the tuning fork tines. Each earphone contained two antiparallel coils, each wrapped around a permanent magnet core. The mounting system and coils are shown in Figure 3. The earphones were mounted so that the polarities of the driver coils were opposite to the polarities of the detector coils. The earphones could be clamped in various locations relative to the tuning fork to allow us to tune the response of the system for optimal performance.

When an alternating current is sent through the driver coil, the oscillating electromagnetic field interacts with the ferromagnetic material in the tuning fork. The resulting vibration of the tines in the electromagnetic field of the detector coil produces eddy currents, which in turn induce current in the detector coil. This system, unfortunately, was very sensitive to background electromagnetic noise. We had to turn off all the fluorescent lights in the lab to minimize the detection of noise. In principle, this system could be used to drive and measure the motion of an aluminum tuning fork, but the response was too small for us to measure.

Measuring the resonant frequency
Given the leisurely pace of this experiment (to avoid thermal lag), we felt free to use a leisurely method of measuring the resonant frequency. We used a variation on the Lissajous figure technique where one channel of an *x-y* oscilloscope is fed a reference frequency and the other channel is fed the signal. If the figure drawn on the face of the oscilloscope is an ellipse that seems to rotate, the two frequencies do not match. The reference frequency then is adjusted to freeze the Lissajous pattern, indicating that the two frequencies are the same. However, the decay of the signal input from the damped tuning fork presents the illusion of rotation in the Lissajous figure, even when the two frequencies are identical. Thus, we had to modify the technique and adapted one usually used for measuring relative phase between two signals. [13]

The oscilloscope is set into a time base mode and the reference signal is used as the time trigger for the sweep. This eliminates effects of a changing reference signal. Figure 4 is a sketch of the oscilloscope face showing the reference signal and the decaying signal produced by the damped tuning fork. The tuning fork initially is set into oscillation by connecting the driver coil to the reference signal. The frequency of the reference signal is set close to the resonant frequency of the tuning fork. The tuning fork quickly builds up a large amplitude of oscillation. The driver coil is then disconnected from the reference signal and the tuning fork allowed to freely oscillate at its natural resonant frequency. If the resonant frequency does not exactly match the reference frequency, the two signals will steadily accumulate a phase shift with respect to each other. This is seen on the oscilloscope face by a drift in the displayed signal from the pick-up coils near the oscillating tuning fork. The frequency of the reference is adjusted to 'freeze' the horizontal drift of the signal from the tuning fork. When the oscilloscope trace is 'frozen', the two frequencies are the same. The limit of the precision of this technique is governed by the precision of the frequency generator [14] and the amount of time that the decaying signal can be monitored before it disappears into the noise. We were able to track the resonant frequency of about 128 Hz as a function of temperature to ±0.003 Hz.

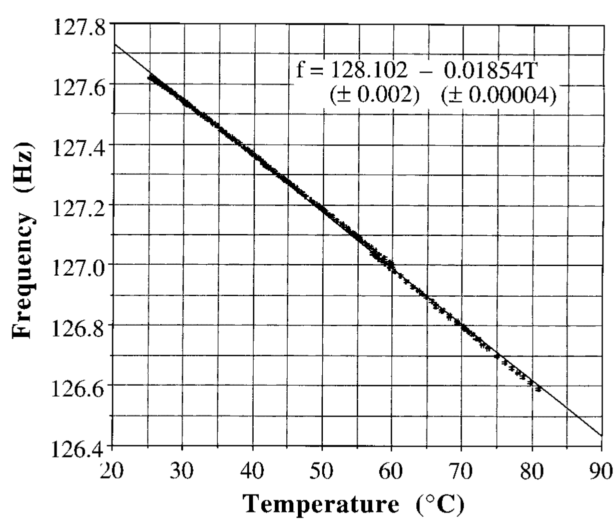

Figure 5
Results of frequency measurements in the temperature range 25 C to 80 C. The graph shows an almost linear dependence with a negative slope.

EXPERIMENTAL RESULTS

Figure 5 shows our measurements of the resonant frequency of the tuning fork as a function of temperature. The frequency vs temperature graph appears almost linear with a slope of -0.01854±0.00004 Hz/C.

The data shown in Figure 5 are transformed into a graph of the relative dependence of Young's modulus using Equation 7. The reference frequency, $f_{n,o}$, was picked as the frequency at 'room temperature' of 18.88 C (a rather chilly lab); 127.727±0.003 Hz. We ignore all thermal expansion factors. Figure 6 shows the measured relative values of Young's modulus as a function of temperature. The temperature dependence is best fit by a quadratic function:

$$\frac{E(T)}{E_o} = (1.00501 \pm 0.00005) - (2.55 \pm 0.02)\text{x}10^{-6}\text{C}^{-1}\, T - (4.5 \pm 0.2)\text{x}10^{-8}\text{C}^{-2}\, T^2, \qquad (9)$$

which is consistent with the nearly linear behavior of the frequency vs temperature graph shown in Figure 5. These results show that Young's modulus for stainless steel has a quadratic dependence on temperature between 20 C and 80 C.

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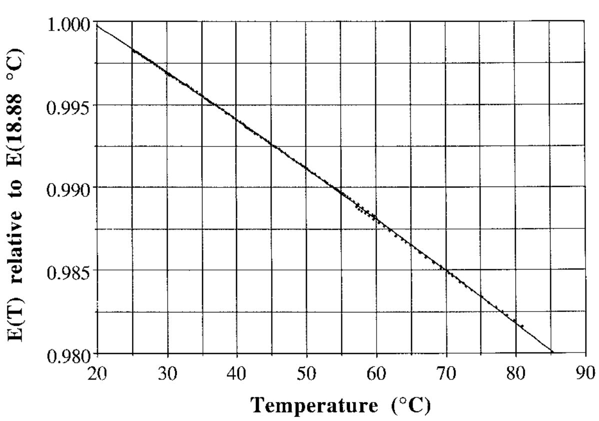

Figure 6
Temperature dependence of Young's modulus expressed as a ratio relative to the value at 18.88 C.

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