

## MODELING PION FLOW IN A $^{139}\text{La} + ^{139}\text{La}$ COLLISION

Daniel Valente \*

Department of Physics and Engineering

Hope College

Holland, MI 49423

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### ABSTRACT

This study focused on the flow behavior of pions created in a  $^{139}\text{La} + ^{139}\text{La}$  nuclear collision. The system was simulated and studied at beam energies of 400 MeV/A and 1200 MeV/A using the Boltzmann-Uehling-Uhlenbeck (BUU) transport model for nuclear collisions. The dependence of flow on centrality, the size of the impact parameter, was examined for both beam energies. The dependence of flow on the nuclear equation of state, as well as on Coulomb interactions, was studied. At 400 MeV/A, both positive and negative pions were found to exhibit flow for central (small) impact parameters and anti-flow for peripheral (large, on the size of the nuclear radius) impact parameters. Similar results were obtained at 1200 MeV/A, agreeing with previous experimental data. The predicted flow was insensitive to the choice of equation of state and the Coulomb interactions did not have any profound change on the pion behavior.

### INTRODUCTION

When two nuclei collide, the interactions between them push nucleons away from the collision zone. Examining how particles flow away from this region provides information on the interactions experienced during the collision. In low-energy collisions, the short-range attractive component of the nuclear force causes the nuclei to rotate and be pulled around each other (Figure 1). In high-energy collisions, the nuclei scatter elastically due to the repulsive hard core of the nuclear force (Figure 1). With this knowledge, the nuclear interaction, or the nuclear equation of state (EoS), can be studied. The EoS is used to model interactions between particles. If we think of particles as being connected by springs, then in a stiff EoS, the particles would be connected by stiff springs; in a soft EoS, the particles would be connected by more flexible springs.

To gain information about the nuclear EoS from the

trajectories of the particles involved in the collision, it is useful to define a physical quantity that describes how these particles are flowing away from the collision zone. In a collision, if forward moving particles (those traveling in the  $+z$  direction) have a component of their momentum in the  $+x$  direction (see Figure 1), then this is defined as *flow*. This is typically seen in the elastic scattering of high-energy collisions. In contrast, if the forward moving particles (those traveling in the  $+z$  direction) have a component of their momentum in the  $-x$  direction (see Figure 1), then this is defined as *anti-flow*. This is characteristic of low-energy collisions, where the particle interactions cause the nuclei to revolve around each other. Thus, on the average, systems with forward moving particles having an average momentum in the  $x$  direction,  $\langle p_x \rangle$ , that is positive exhibit *flow*, while systems with forward moving particles having a negative  $\langle p_x \rangle$  exhibit *anti-flow*.

For particles moving backwards in the center-of-momentum (CM) frame, the opposite is true; backward moving particles having a positive  $\langle p_x \rangle$  exhibit anti-flow, and backward moving particles having a negative  $\langle p_x \rangle$  exhibit flow. The momenta of the particles in these directions also determine the strength of the flow—the greater the momenta, the greater the flow. To determine the directionality of the particle motion (forward or backward), the particle rapidity  $y$  is used:

$$y = \ln \left[ \frac{E + p_z}{E - p_z} \right]. \quad (1)$$

*Dan graduated from Hope College in May 2001 with a Bachelor of Science in Physics. This research was begun in the summer of his sophomore years as part of a NSF Research Experience for Undergraduates and was independently continued in his senior year. Dan is currently attending the graduate program in acoustics at the Pennsylvania State University, and is working towards his Ph.D. In his free time, Dan enjoys song writing, playing the guitar and drawing.*

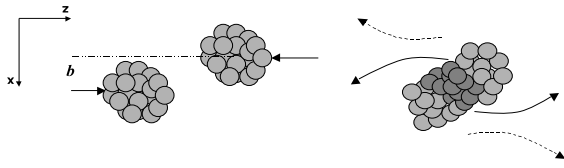


Figure 1

This figure shows the time progression of a nuclear collision. The two nuclei on the left side of the figure shows the system before the collision, where  $b$  is the impact parameter of the collision. The right side of the figure shows the system after the collision. The darkly shaded region is the hot region where pions are produced. The solid lines represent the trajectories of single particles exhibiting anti-flow after the collision. The dotted lines represent the trajectories of single particles exhibiting flow after the collision. Note that the  $x$ -direction points towards the bottom of the page.

One might expect that the velocity in the beam direction ( $v_z$ ) would be used for this purpose, but rapidity is used in place of velocity for ease in transformations from the lab frame to the CM frame. Rapidity transforms linearly as:

$$y' = y_o + y_{frame}, \quad (2)$$

instead of the complicated Einstein transformation needed for velocity. Together, both  $y$  and  $\langle p_x \rangle$  give an informative description of the directionality of particle movement.

Typically, flow is examined on a plot of  $\langle p_x \rangle$  versus  $y$ . The resulting graphs have a characteristic S-shaped curve to them, as seen in Figure 2. An inspection of the slope of the curve at the origin gives us a quantitative evaluation for flow. Forward moving particles with a positive  $\langle p_x \rangle$  produce a positive slope about the origin, indicating flow. Forward moving particles with a negative  $\langle p_x \rangle$  produce a negative slope, which is termed anti-flow. The steepness of the slope gives information on the amount of flow a system is exhibiting—the larger the slope, the greater the flow. A more extensive explanation of flow in nuclear reactions can be found in the literature. <sup>1,2</sup>

At high energies, nucleons in a collision always produce flow plots similar to Figure 2; they always exhibit flow. In energetic collisions, however, nucleons are not the only particles involved. Around the area of impact, a region of high-density nuclear matter is created. Conditions in this hot region are acceptable for the creation of a variety of particles. If the energy in the hot zone is above production threshold, a nucleon-nucleon collision can create a  $\Delta$  particle. The  $\Delta$  particle is a baryon with a lifetime of  $\sim 10^{-23}$  s that decays into a nucleon by emission of a pion (pions are mesons consisting of a quark/anti-quark pair). A thorough discussion of meson production in nuclear collisions can be found elsewhere. <sup>3</sup>

A search of the literature found an experiment that examined an Au + Au system at 1.15 GeV, which was energetic enough to create  $\Delta$  particles and pions. <sup>4</sup> It was found for this energy that pion flow is dependent upon impact parameter. For peripheral collisions (collisions in which the edges of the nuclei collide), positive pions and negative pions exhibit a flow that is in a direction opposite to that of the nucleons. This was attributed to shadowing and re-scattering effects of the spectator nucleons. Shadowing occurs when the residual nuclei block (shadow) the pions from reaching the detectors. Re-scattering occurs when the pions interact with the other particles involved in the collision and are scattered multiple times.

In this work, a computer model was used to simulate nuclear collisions, and the resulting flow behavior was examined. The goal of these simulations was to understand the experimental results. <sup>4</sup> Understanding why pions flow in a certain direction elucidates the properties of the hot zone and the interactions that pions experience in this region, thus providing vital information about the system.

#### THEORETICAL EXAMINATION OF THE $^{139}\text{La} + ^{139}\text{La}$ COLLISION: BUU

In this study, a  $^{139}\text{La} + ^{139}\text{La}$  collision was modeled using the Boltzmann-Uehling-Uhlenbeck (BUU) transport model for nuclear collisions to examine pion flow. BUU simulated the collision of two nuclei by using Monte-Carlo techniques to track the trajectories of nucleons of the colliding nuclei as they move under the influence of a nuclear potential  $U$ . <sup>5</sup> These nucleons have randomly generated initial positions constrained within a sphere the size of the nucleus and a randomly generated momentum

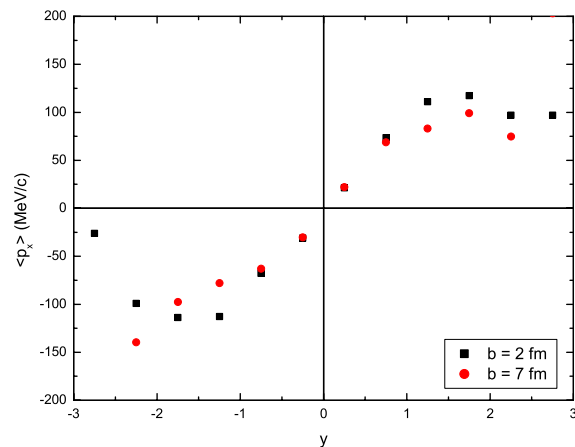


Figure 2

Typical flow curves for nucleons in 1200 MeV/A La + La collisions at impact parameters of 2 fm and 7 fm for a stiff EoS. The well defined S-curve is characteristic of flow plots.

constrained to be below the Fermi momentum for the ground state of the nucleus. At each time step, (the size of the time step is a parameter of the model), BUU solves a form of Newton's second law for every particle involved in the collision (including particles created in the collision), allowing their momenta and positions to be tracked as a function of time. The model takes into account the probabilities for particle-particle scattering using measured reaction cross sections, as well as the effects of the Pauli exclusion principle, forbidding collisions in which the final states of the scattered particles are occupied.

Since  $U$  describes the nuclear interaction, it inherently contains information about the nuclear equation of state. In effect, BUU tracks the change in the number of particles that have a certain momentum at a certain position from the original randomly generated state to the final state caused by the interactions in the collision. An examination of this change yields information about the interactions described by the nuclear force (the EoS).

The energy of the collision and the impact parameter should have an effect on the flow of the particles. Since *flow* depends on the complex interactions experienced by the particles, perhaps the EoS can be probed by examining the *flow* behavior of pions in a collision. To test this, two separate impact parameters (central and peripheral) were examined at two beam energies (1200 MeV/A and 400 MeV/A) in a Lanthanum/Lanthanum system. A beam energy per nucleon (MeV/A) of 1200 MeV/A was run to compare to the experimental data<sup>4</sup>. A lower beam energy of 400 MeV/A was run to compare to the higher energy. Central collisions (those which are nearly head-on) were defined by an impact parameter of  $b = 2$  fm. Peripheral collisions were defined as  $b = 7$  fm, which is slightly larger than the radius of the Lanthanum nucleus.

The nuclear interaction is a parameter of the model as well, so two nuclear EoS, one stiff, momentum independent and one soft, momentum dependent, were examined to see if the definition of the EoS gave characteristic differences in *flow*. (In a momentum dependent EoS the trajectories of each particle are influenced by the momentum of the particles with which it interacts). Finally, to study the effects of the Coulomb interaction on *flow*, simulations were run with the effects of the Coulomb potential neglected. These simulations were also used to examine any differences between the *flow* behavior of positive and negative pions.

## RESULTS

Figure 3 shows the results obtained for negative pions at 400 MeV/A for the stiff EoS. Here, a distinct difference can be seen in pion *flow* at different impact parameters. For the central ( $b = 2$  fm) impact parameter, negative pions show signs of *flow* (positive slope about the origin), whereas in peripheral ( $b = 7$  fm) impacts, the negative pions show signs of *anti-flow* (negative slope about the origin). The same is true for positive pions at the same

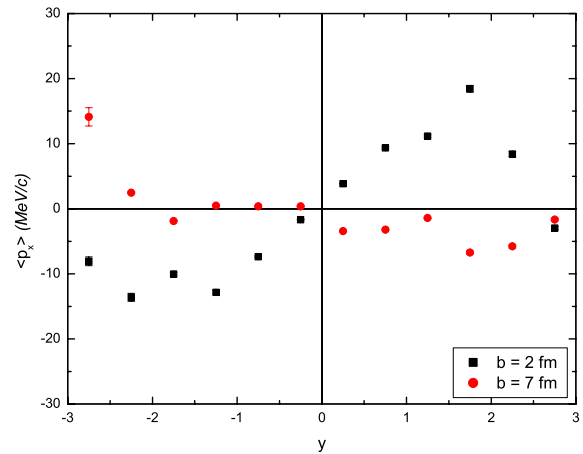


Figure 3

The flow curves for negative pions in 400 MeV/A La + La collisions at impact parameters of 2 fm and 7 fm for a stiff EoS. Here the impact parameter dependence is evident.

energy.

To gain insight on the impact parameter dependence, the development of *flow* over time is examined in Figure 4. In Figure 4, *flow* as a function of  $\pi^-$  emission time is plotted. Note that in Figure 4, unlike the *flow* plots, the y-axis is a weighted transverse momentum  $\langle wp_x \rangle$  (MeV/c). Because of the symmetry of the collision, a particle with momentum  $p_x$  and rapidity  $y_1$  going backward can be mapped to a particle with momentum  $-p_x$  and rapidity  $-y_1$ . Thus, the weighting factor  $w$  is +1 for  $y > 0$  and -1 for  $y < 0$  so that the statistics in these calculations can be doubled. The x-axis is time in fm/c ( $\sim 0.3 \times 10^{-23}$  s). At both impact parameters, the pions emitted in early times of the collision exhibit *flow*. As the collision progresses, the peripheral collisions show a transition to *anti-flow*. The peak in *anti-flow* happens at a time of approximately 35 fm/c. A summary of all the *flow* results can be found in Table 1.

As can be seen in Table 1, both positive and negative pions behave similarly in all of the simulations. Because of the similarity, it is assumed that this *flow* behavior is a result of the geometry of the collision, rather than a Coulomb effect. A possible explanation for the peak in *anti-flow* for peripheral collisions follows directly from an investigation of Figure 4. When two nuclei collide in a central collision, the hot zone occurs in the middle of the large nucleonic mass that is subsequently formed. The pions created here have a difficult time escaping this central region, as they are re-absorbed by the residual nuclei. At all times in the collision, these pions are "carried" with the collection of nucleons and exhibit the same *flow* characteristics as the nucleons (Figure 5a). In peripheral collisions, however, the pions are not trapped in this large central region; they are created in a narrow neck region around the area of

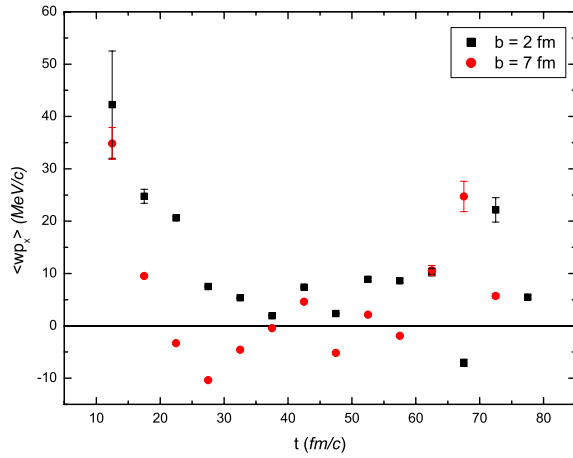


Figure 4

The time dependence of flow for negative pions in 400 MeV/A La + La collisions at impact parameters of 2 fm and 7 fm for a stiff EoS. This figure shows  $\langle v_{p_x} \rangle$  at the times the pions are emitted from the system. Integration of these curves over time yield Figure 3. At both impact parameters, the system shows flow at early times in the collision, but the peripheral show a transition to anti-flow at  $\sim 35$  fm/c, suggesting a geometric effect.

impact. As the nuclei pass each other, the pions are forced in the direction of flow; any anti-flow path is blocked by the residual nuclei. However, when the nuclei begin to pass each other (at a time around 30-35 fm/c) the geometry of the collision shifts and the flow path is now blocked by the nuclei, thus causing the pions to exhibit anti-flow (see Figure 5b).

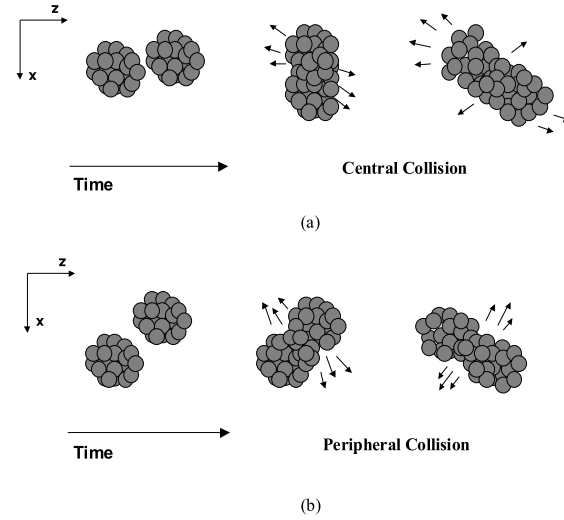


Figure 5

Pictorial representation describing the geometric effect suggested by Figure 3. The black arrows represent the paths of the pions. a) In central collisions, the pions are created in a region from which they have difficulty escaping. Therefore, they travel with the nucleons, exhibiting flow. b) In peripheral collisions the pions are created in a narrow neck region. The nucleons force flow at early times in the collision, but block this path at later times, forcing pion anti-flow.

At 1200 MeV/A with a stiff EoS, results similar to the lower energy data are obtained. As seen in Figure 6, the impact parameter dependence of flow is still apparent.

	Central Collisions ( $b = 2$ fm)		Peripheral Collisions ( $b = 7$ fm)	
	Stiff EoS	$p$ dependent EoS	Stiff EoS	$p$ dependent EoS
400 MeV/A				
Positive Pions	13.72	15.77	-3.561	*
Negative Pions	11.15	10.54	-1.381	*
Protons	115.9	101.5	86.20	110.3
Neutrons	100.1	90.56	76.28	96.68
1200 MeV/A				
Positive Pions	*	*	-17.54	-19.42
Negative Pions	5.720	4.465	-20.43	-16.94
Protons	94.51	93.17	89.46	96.38
Neutrons	85.04	84.97	89.30	98.98
	*slope close to zero			

Table 1

A summary of the flow results showing the slope of the fit line to the curve about the origin (the flow) for each simulation. The magnitude of the slope describes the degree to which particles are showing flow or anti-flow.

Central collisions exhibit *flow*, whereas peripheral collisions exhibit *anti-flow*. This is consistent with the experimental data. Figure 7 shows the *flow* as a function of time. These results for the higher energy differ slightly from those at the lower energy. At the high energy, less *flow* and a larger *anti-flow* are observed. Because the 1200 MeV/A collision is much more energetic, it happens on a much faster time scale. Therefore, the system never spends much time in a geometry that forces *flow*. Pions created in central collisions are still reabsorbed by the residual nuclei, but the pions created in peripheral collisions are emitted when the nuclei have nearly passed each other.

Because the pions are charged particles, Coulomb effects on the *flow* were examined in simulations that neglected Coulomb effects. When the data from these simulations were compared with the previous simulations, a small effect on *flow* due to the Coulomb interaction was found. For positive pions in central collisions, the Coulomb repulsion augments *flow*, since these pions are emitted with momentum that is already in the *flow* directions. In peripheral collisions, the geometry of the system causes the interaction to push the pions towards *anti-flow*. For negative pions at central collisions, the Coulomb interaction has the effect of adding to the *flow*, since these pions are attracted to the nucleons. For peripheral collisions, this attraction subtracts from *anti-flow*. In no case did the Coulomb interaction drastically change the *flow* or *anti-flow*, but it did slightly enhance it.

Lastly, no significant differences were found between the results for the stiff, momentum independent EoS (characteristic of the above simulations) and the soft, momentum

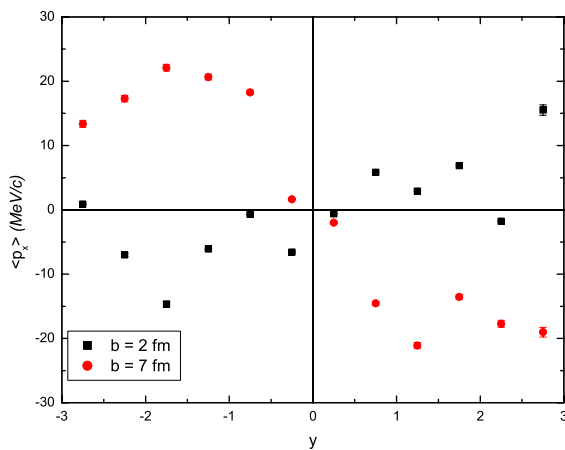


Figure 6

The flow curves for negative pions in 1200 MeV/A La + La collisions at impact parameters of 2 fm and 7 fm for a stiff EoS

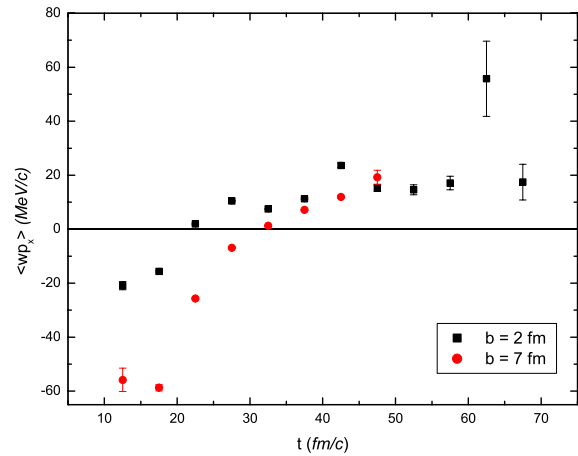


Figure 7

The time dependence of flow for negative pions in 1200 MeV/A La + La collisions at impact parameters of 2 fm and 7 fm for a stiff EoS

dependent EoS. The *flow* characteristics of all particles were nearly the same (see Table 1).

## SUMMARY

The experimental evidence of the impact parameter dependence for *flow* has been confirmed by this study, however, it is apparent from the results in Table 1 that pion *flow* is not dependent on the nuclear EoS. It has been hypothesized that a geometric effect may be the cause of the impact parameter dependence, as well as the strength of the *flow*. This hypothesis has been supported by findings at two different beam energies. Further studies should focus on different parameters of the collision to confirm this geometric effect as well as investigations on what parameters can yield the most useful information about the equation of state.

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Dr. Catherine M. Mader  
Department of Physics and Engineering  
Hope College  
Holland, MI 49423  
mader@physics.hope.edu